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COMMENT

Comment on 'Irreducible Green function theory for ferromagnets with first- and second-neighbour exchange'

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Abstract. The 'new' irreducible Green function theory suggested by Mitra and Chakraborty (*J. Phys.: Condens. Matter* 7 (1995) 379) is shown to be inconsistent and incorrect.

The Heisenberg model Hamiltonian with first- and second-neighbour interactions is given by

$$H = \mu h \sum_i S_i^z - \sum_{i_1 j_1} J^{(1)}_{i_1 j_1}(S_{i_1} \cdot S_{j_1}) - \sum_{i_2 j_2} J^{(2)}_{i_2 j_2}(S_{i_2} \cdot S_{j_2}) \tag{1}$$

where $\{S_{i_1}^{\mu}\}$, $\mu = x, y, z$, are the components of a spin operator at site i_1, $J^{(1)}_{i_1 j_1}$ and $J^{(2)}_{i_2 j_2}$ are, respectively, the nearest- and next-nearest-neighbour exchange integrals respectively, and the first term represents the coupling of the system to an external z field. The central parameter in the standard irreducible Green function (SIRG) treatment of (1) is the residual momentum space Zubarev Green function (Mitra and Chakraborty 1995),

$$\langle\langle \Phi_{k,k'}; S_q^- \rangle\rangle \equiv \left\langle\left\langle \left(S_{k'}^z S_{k-k'}^+ - S_{k-k'}^z S_{k'}^+\right) - C_{mf} - C_{emf}; S_q^- \right\rangle\right\rangle \tag{2}$$

where C_{mf} and C_{emf} are given by

$$C_{mf} \equiv \langle S_{k'}^z \rangle S_{k-k'}^+ - \langle S_{k-k'}^z \rangle S_{k'}^+ \tag{3}$$

$$C_{emf} \equiv \left(A_{k'} - A_{k-k'}\right) S_k^+ . \tag{4}$$

In the SIRG approach, A_k is determined by the irreducibility condition

$$\lambda_{k,k',q} \equiv 0 \tag{5}$$

$$\lambda_{k,k',q} \equiv \left\langle\left[\Phi_{k-k'},\, S_q^-\right]\right\rangle . \tag{6}$$

Mitra and Chakraborty (1995) have suggested that the irreducibility condition (5), may be replaced by the less restrictive condition that $\lambda_{k,k',q}$ is independent of k'. They have shown that, if $\lambda_{k,k',q}$ is independent of k', it disappears from the relevant Green function equation of motion. They have asserted that the assumed k' independence allows one to choose *any* A_k (they chose A_k in accordance with the Callen decoupling scheme) since A_k no longer must be chosen to satisfy the irreducibility condition, (5).

The purpose of this note is to show that if $\lambda_{k,k',q}$ is independent of k', then it must vanish and A_k must still satisfy the irreducibility condition (5). The proof is straightforward. If $\lambda_{k,k',q}$ is independent of k', it must have the same value for all k'. In particular, we must have

$$\lambda_{k,k',q} = \lambda_{k,k'=0,q} = \lambda_{k,k'=k,q}. \tag{7}$$

However, from (2), (3) and (4), one readily obtains

$$\lambda_{k,k'=0,q} = -\lambda_{k,k'=k,q} \tag{8}$$

so that (7) gives

$$\lambda_{k,k',q} = 0 \tag{9}$$

if $\lambda_{k,k',q}$ is independent of k'. Thus, A_k must still be chosen to satisfy (9) and, in particular, cannot be chosen in accordance with the Callen decoupling scheme. Therefore, the Mitra–Chakraborty treatment of (1) is inconsistent and incorrect.

References

Mitra S N and Chakraborty K G 1995 *J. Phys.: Condens. Matter* 7 379